Graph Theory in Operational Research

Application exercise 1

Crossing Paris

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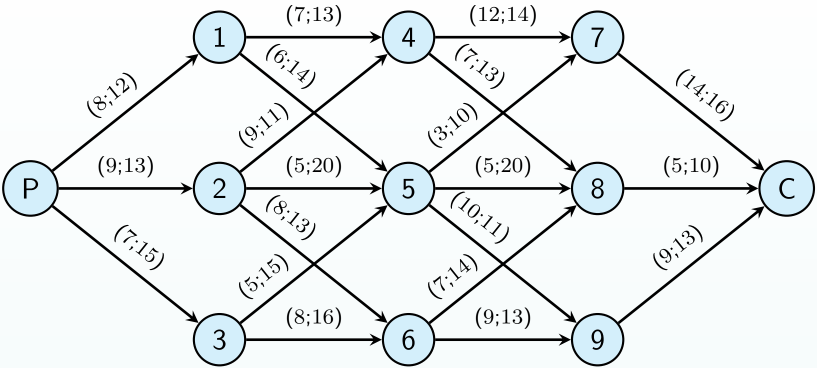
Crossing Paris

- A working shuttle makes the journey every day from Porte d'Orléans to la Chapelle
- 9 squares are identified as many possible crossings points
- Between each of them, the minimum and maximum travel time are known, depending on traffic hazards
- The manager of the company seeks to optimize the shuttle route

Crossing Paris



1. How many possible paths? *(explain the method)*
2. What is the worst path in the worst case? *(write the algorithm)*
3. What is the most optimistic path? Calculate the variation margin. *(present the problem before solving)*
4. What is the most careful path? Calculate the variation margin.
5. What is the most stable path? Calculate the variation margin.
6. Compare the 3 options and their variation margin.

Number of paths: matrix solution

- We know that:
  - for M, matrix successor of G, $M^{\alpha}_{(i,j)}$ is the number of unique path with a length of α from i to j;
  - without cycles, $\exists n_0, \forall n \geq n_0, M^{n}_{(i,j)} = 0$;
- There is thus a finite sum of $\sum_{n=1}^{n_0} M^{n}_{(P,C)}$, set of paths from P to C.

Number of paths: matrix solution

$M =$

	P	1	2	3	4	5	6	7	8	9	C
P	–	1	1	1	–	–	–	–	–	–	–
1	–	–	–	–	1	1	–	–	–	–	–
2	–	–	–	–	1	1	1	–	–	–	–
3	–	–	–	–	–	1	1	–	–	–	–
4	–	–	–	–	–	–	–	1	1	–	–
5	–	–	–	–	–	–	–	1	1	1	–
6	–	–	–	–	–	–	–	–	1	1	–
7	–	–	–	–	–	–	–	–	–	–	1
8	–	–	–	–	–	–	–	–	–	–	1
9	–	–	–	–	–	–	–	–	–	–	1
C	–	–	–	–	–	–	–	–	–	–	–

$$M_{(P,C)} = 0$$

Number of paths: matrix solution

$M^2 =$

	P	1	2	3	4	5	6	7	8	9	C
P	–	–	–	–	2	3	2	–	–	–	–
1	–	–	–	–	–	–	–	2	2	1	–
2	–	–	–	–	–	–	–	2	3	2	–
3	–	–	–	–	–	–	–	1	2	2	–
4	–	–	–	–	–	–	–	–	–	–	2
5	–	–	–	–	–	–	–	–	–	–	3
6	–	–	–	–	–	–	–	–	–	–	2
7	–	–	–	–	–	–	–	–	–	–	–
8	–	–	–	–	–	–	–	–	–	–	–
9	–	–	–	–	–	–	–	–	–	–	–
C	–	–	–	–	–	–	–	–	–	–	–

$M_{(P,C)} = 0$; $M^2_{(P,C)} = 0$

Number of paths: matrix solution

$M^3 =$

	P	1	2	3	4	5	6	7	8	9	C
P	–	–	–	–	–	–	–	5	7	5	–
1	–	–	–	–	–	–	–	–	–	–	5
2	–	–	–	–	–	–	–	–	–	–	7
3	–	–	–	–	–	–	–	–	–	–	5
4	–	–	–	–	–	–	–	–	–	–	–
5	–	–	–	–	–	–	–	–	–	–	–
6	–	–	–	–	–	–	–	–	–	–	–
7	–	–	–	–	–	–	–	–	–	–	–
8	–	–	–	–	–	–	–	–	–	–	–
9	–	–	–	–	–	–	–	–	–	–	–
C	–	–	–	–	–	–	–	–	–	–	–

$$M_{(P,C)} = 0 \; ; \; M^2_{(P,C)} = 0 \; ; \; M^3_{(P,C)} = 0$$

Number of paths: matrix solution

$M^4 =$

	P	1	2	3	4	5	6	7	8	9	C
P	–	–	–	–	–	–	–	–	–	–	17
1	–	–	–	–	–	–	–	–	–	–	–
2	–	–	–	–	–	–	–	–	–	–	–
3	–	–	–	–	–	–	–	–	–	–	–
4	–	–	–	–	–	–	–	–	–	–	–
5	–	–	–	–	–	–	–	–	–	–	–
6	–	–	–	–	–	–	–	–	–	–	–
7	–	–	–	–	–	–	–	–	–	–	–
8	–	–	–	–	–	–	–	–	–	–	–
9	–	–	–	–	–	–	–	–	–	–	–
C	–	–	–	–	–	–	–	–	–	–	–

$$M_{(P,C)} = 0 \; ; \; M^2_{(P,C)} = 0 \; ; \; M^3_{(P,C)} = 0 \; ; \; M^4_{(P,C)} = 17$$

Number of paths: matrix solution

$M^5 =$

	P	1	2	3	4	5	6	7	8	9	C
P	–	–	–	–	–	–	–	–	–	–	–
1	–	–	–	–	–	–	–	–	–	–	–
2	–	–	–	–	–	–	–	–	–	–	–
3	–	–	–	–	–	–	–	–	–	–	–
4	–	–	–	–	–	–	–	–	–	–	–
5	–	–	–	–	–	–	–	–	–	–	–
6	–	–	–	–	–	–	–	–	–	–	–
7	–	–	–	–	–	–	–	–	–	–	–
8	–	–	–	–	–	–	–	–	–	–	–
9	–	–	–	–	–	–	–	–	–	–	–
C	–	–	–	–	–	–	–	–	–	–	–

$M_{(P,C)} = 0$; $M^2_{(P,C)} = 0$; $M^3_{(P,C)} = 0$; $M^4_{(P,C)} = 17$;
$M^5_{(P,C)} = 0$

Number of paths: recursive solution

- Le number of paths from x to y is equal to the sum of paths from the successors of x and y.

$$
\begin{aligned}
&P(P) = P(1) + P(2) + P(3)\\
&P(1) = P(4) + P(5)\\
&P(2) = P(4) + P(5) + P(6)\\
&P(3) = P(5) + P(6)\\
&P(4) = P(7) + P(8)\\
&P(5) = P(7) + P(8) + P(9)\\
&P(6) = P(8) + P(9)\\
&P(7) = P(C)\\
&P(8) = P(C)\\
&P(9) = P(C)\\
&P(C) = 1
\end{aligned}
$$

Number of paths: recursive solution

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$$P(P) = P(1) + P(2) + P(3)$$
$$P(1) = P(4) + P(5)$$
$$P(2) = P(4) + P(5) + P(6)$$
$$P(3) = P(5) + P(6)$$
$$P(4) = P(7) + P(8)$$
$$P(5) = P(7) + P(8) + P(9)$$
$$P(6) = P(8) + P(9)$$
$$P(7) = P(C) = 1$$
$$P(8) = P(C) = 1$$
$$P(9) = P(C) = 1$$
$$P(C) = 1$$

Number of paths: recursive solution

- Le number of paths from x to y is equal to the sum of paths from the successors of x and y.

$$P(P) = P(1) + P(2) + P(3)$$
$$P(1) = P(4) + P(5)$$
$$P(2) = P(4) + P(5) + P(6)$$
$$P(3) = P(5) + P(6)$$
$$P(4) = P(7) + P(8) = 2$$
$$P(5) = P(7) + P(8) + P(9) = 3$$
$$P(6) = P(8) + P(9) = 2$$
$$P(7) = P(C) = 1$$
$$P(8) = P(C) = 1$$
$$P(9) = P(C) = 1$$
$$P(C) = 1$$

Number of paths: recursive solution

- Le number of paths from x to y is equal to the sum of paths from the successors of x and y.

$$P(P) = P(1) + P(2) + P(3)$$
$$P(1) = P(4) + P(5) = 5$$
$$P(2) = P(4) + P(5) + P(6) = 7$$
$$P(3) = P(5) + P(6) = 5$$
$$P(4) = P(7) + P(8) = 2$$
$$P(5) = P(7) + P(8) + P(9) = 3$$
$$P(6) = P(8) + P(9) = 2$$
$$P(7) = P(C) = 1$$
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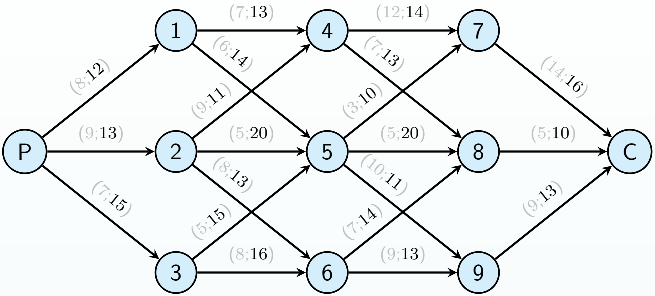
$$P(C) = 1$$

Number of paths: recursive solution

- Le number of paths from x to y is equal to the sum of paths from the successors of x and y.

$$P(P) = P(1) + P(2) + P(3) = 17$$
$$P(1) = P(4) + P(5) = 5$$
$$P(2) = P(4) + P(5) + P(6) = 7$$
$$P(3) = P(5) + P(6) = 5$$
$$P(4) = P(7) + P(8) = 2$$
$$P(5) = P(7) + P(8) + P(9) = 3$$
$$P(6) = P(8) + P(9) = 2$$
$$P(7) = P(C) = 1$$
$$P(8) = P(C) = 1$$
$$P(9) = P(C) = 1$$
$$P(C) = 1$$

Worst path in worst case



$\lambda_1 \leftarrow 0; M \leftarrow \{x_1\}$

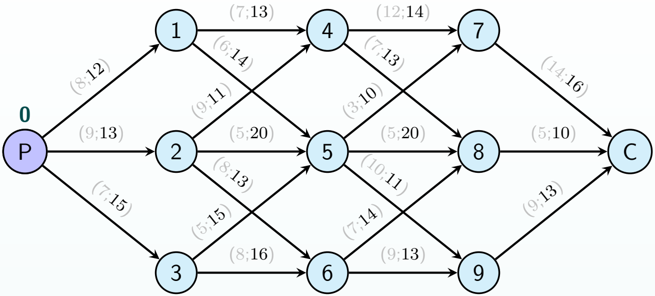
while $M \neq X$, do

  select $x_j \in X \setminus M$ such as $P(x_j) \subset M$

  $\lambda_j \leftarrow \max\limits_{i:x_i \in P_{x_j}} \{\lambda_i + v_{ij}\}$

  $M \leftarrow M \cup \{x_j\}$

Worst path in worst case



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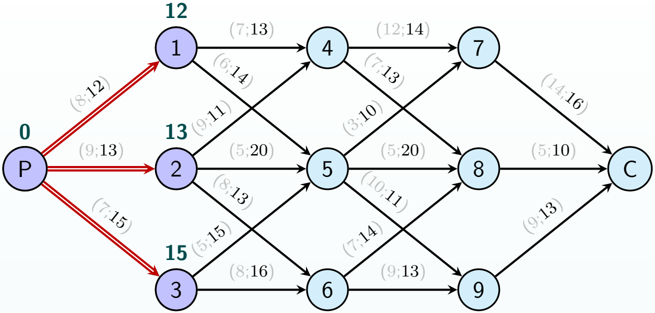

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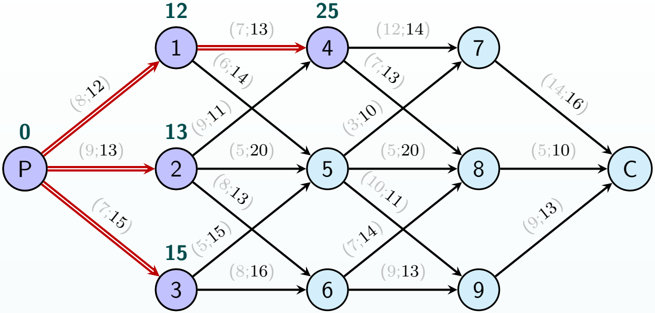

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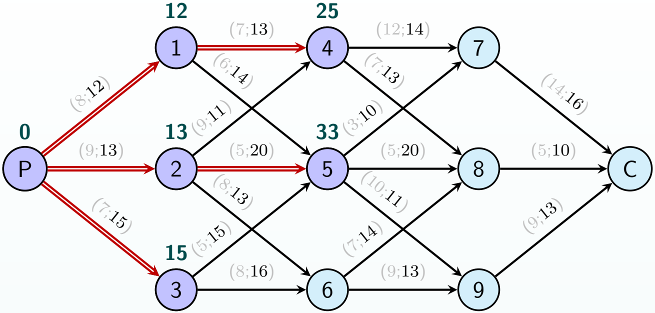

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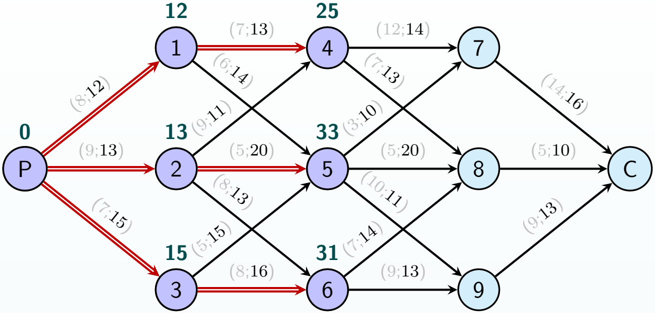

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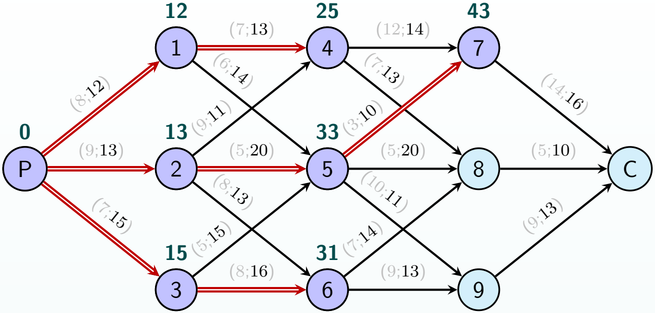

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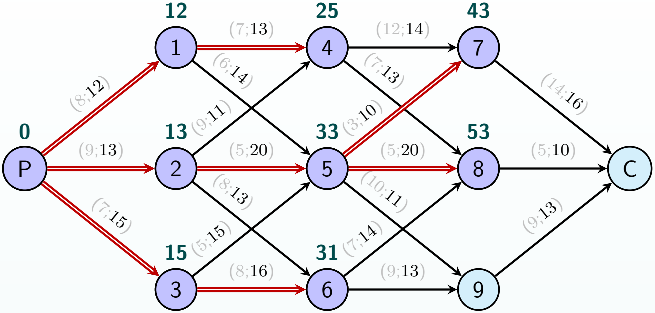

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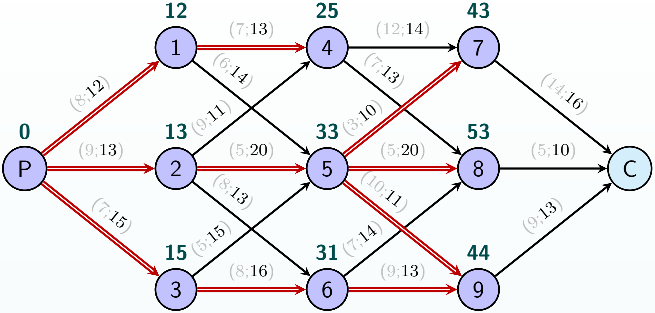

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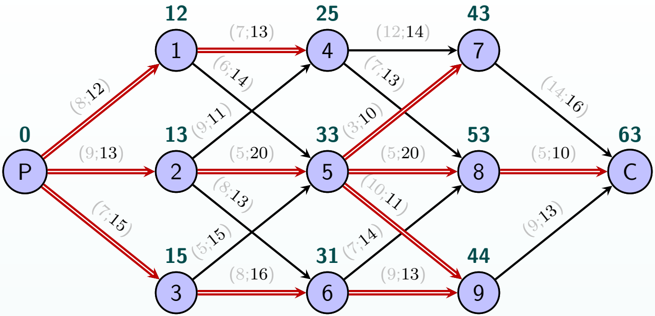

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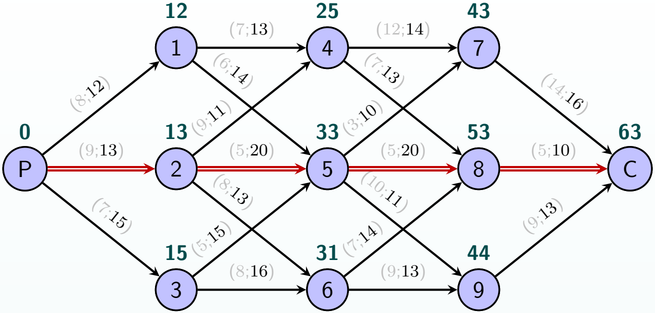

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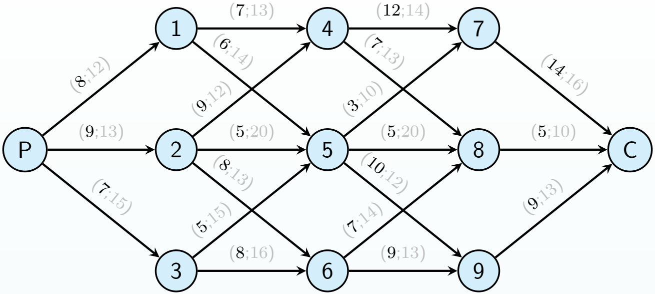
Optimistic path



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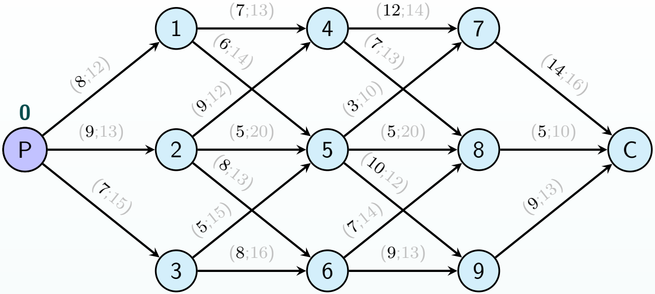

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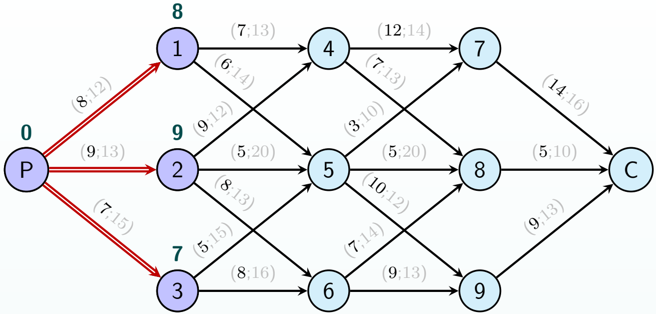

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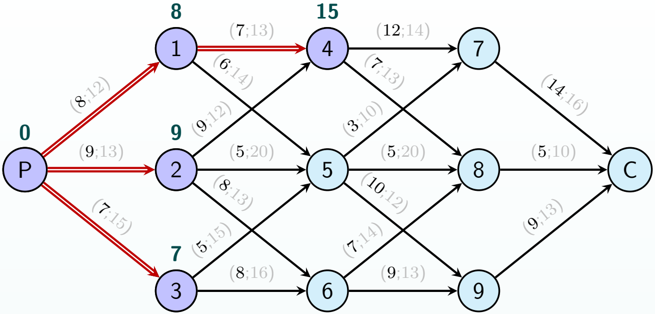

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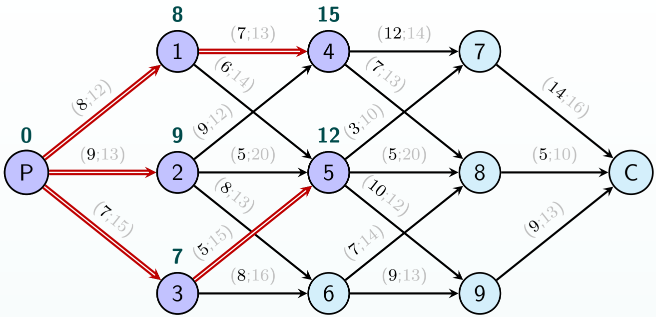

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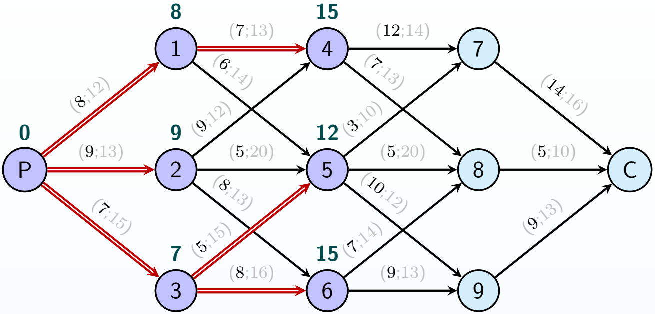

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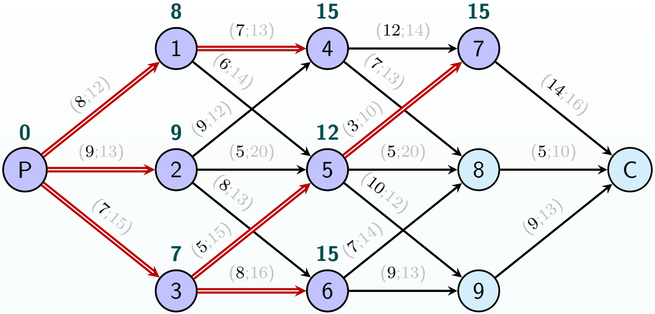

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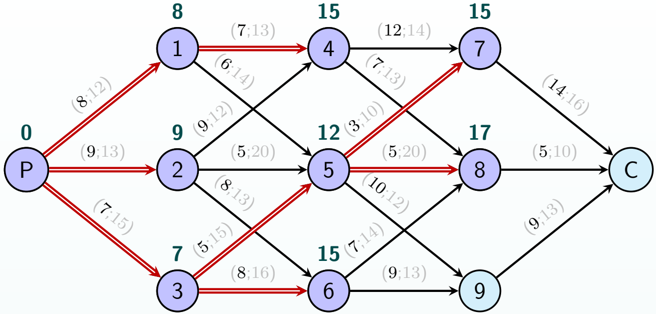

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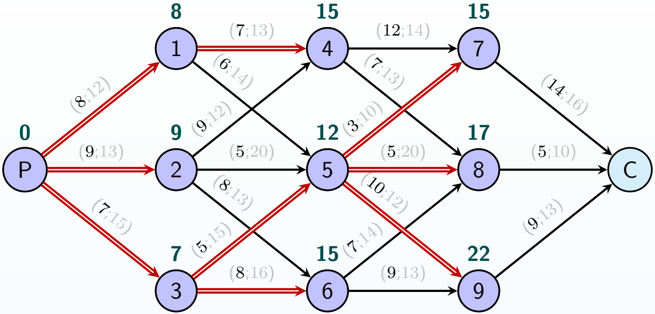

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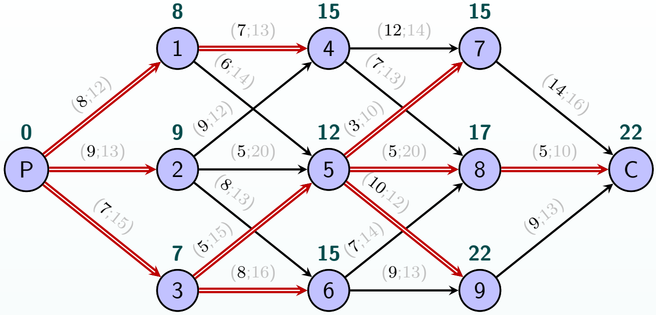
Optimistic path



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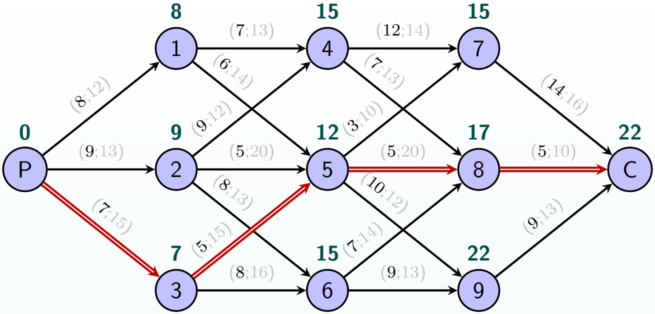

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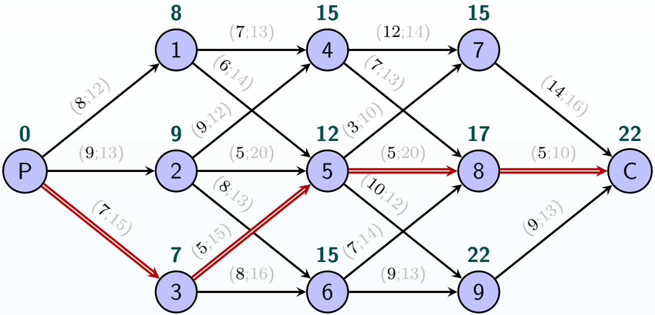
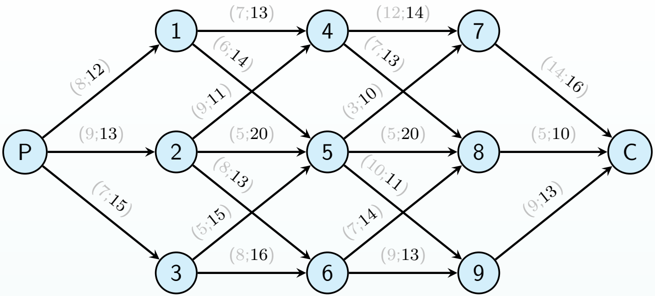
Optimistic path



- variation margin:
  $m(\mu^{\text{opt}}) = d_{\text{max}}(\mu^{\text{opt}}) - d_{\text{min}}(\mu^{\text{opt}}) = 60 - 22 = 38$

Careful path



$\lambda_1 \leftarrow 0; M \leftarrow \{x_1\}$

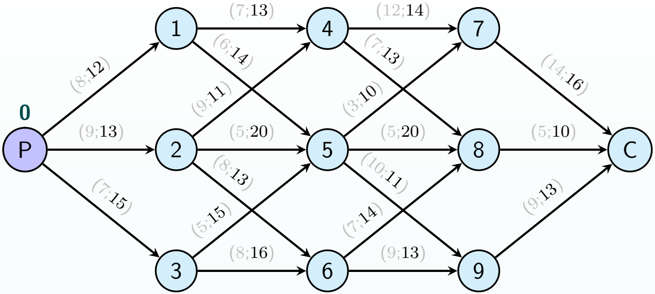
while $M \neq X$, do

  select $x_j \in X \setminus M$ such as $P(x_j) \subset M$

  $\lambda_j \leftarrow \min_{i:x_i \in P_{x_j}} \{\lambda_i + v_{ij}\}$

  $M \leftarrow M \cup \{x_j\}$

Careful path



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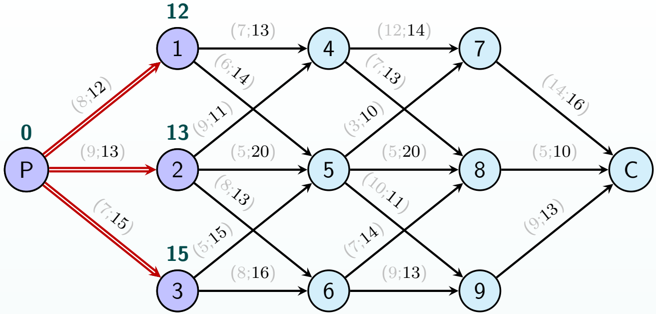
Careful path



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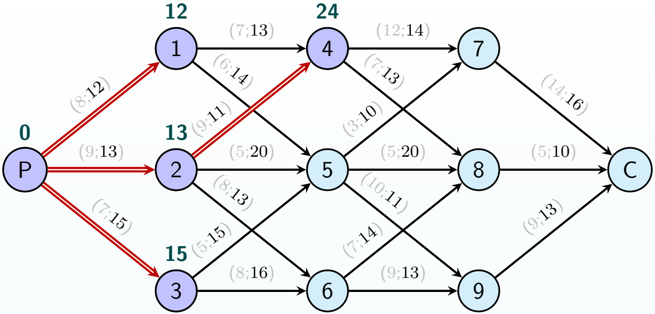
Careful path



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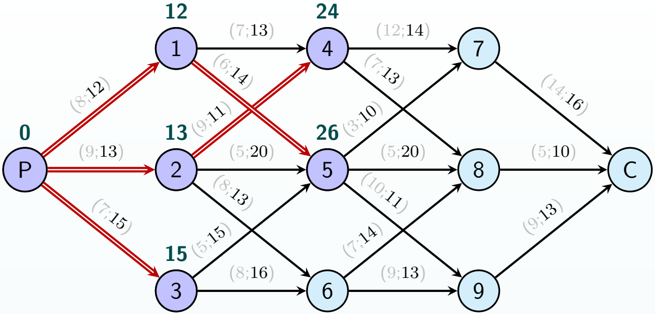
Careful path



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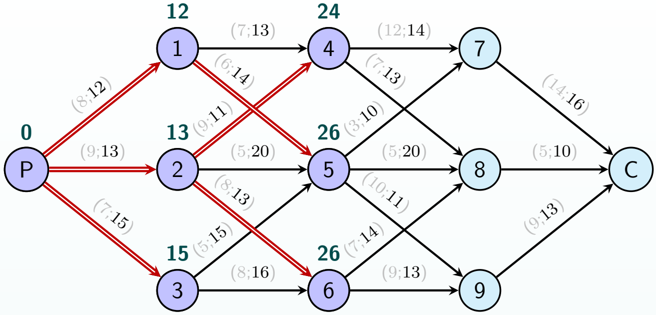
Careful path



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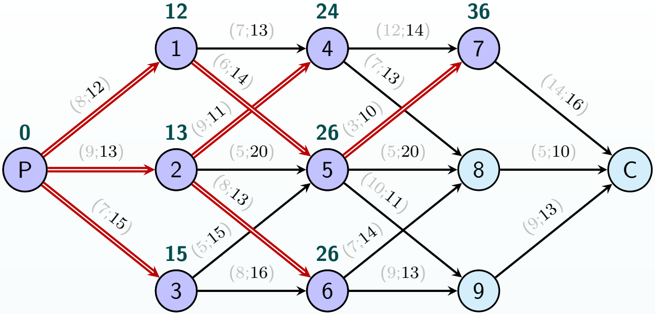

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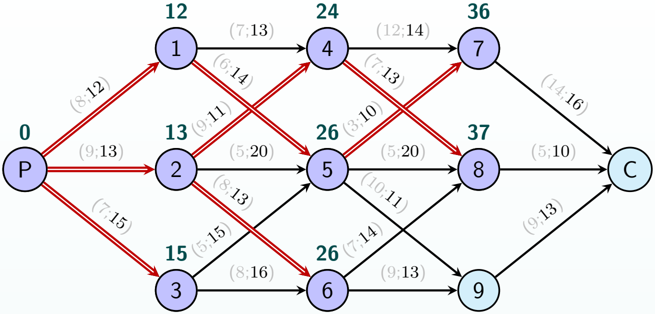

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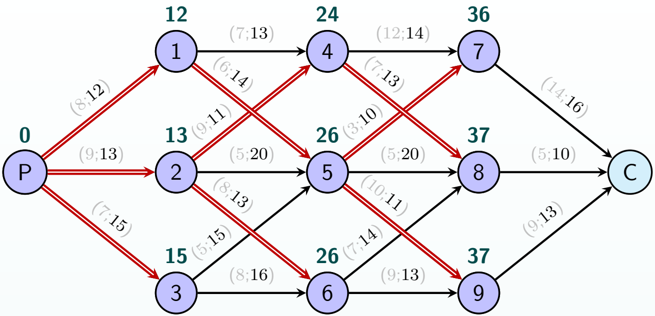
Careful path



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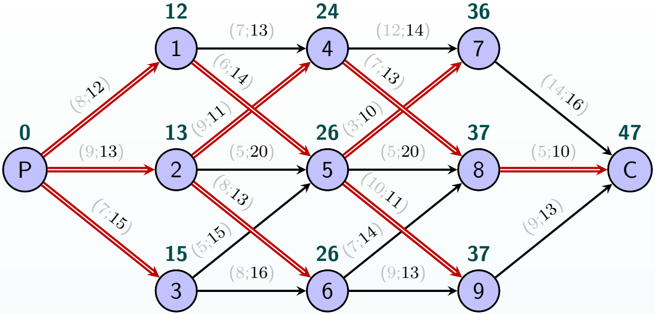

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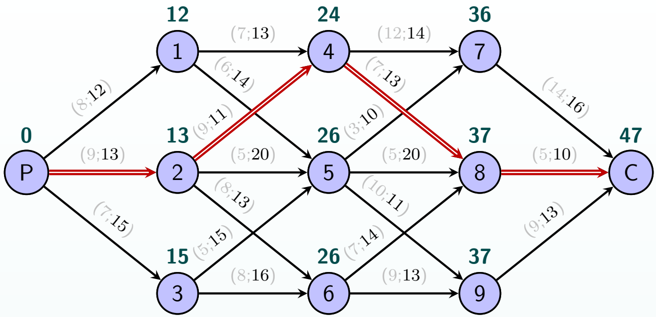
Careful path



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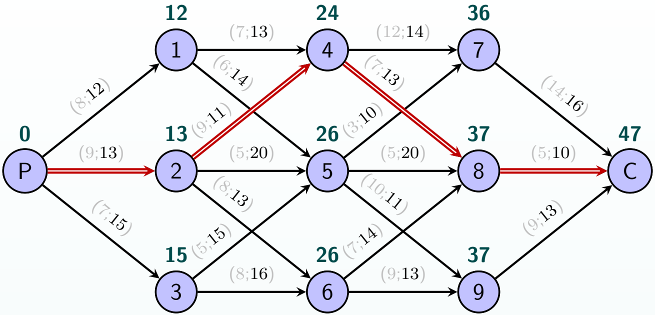
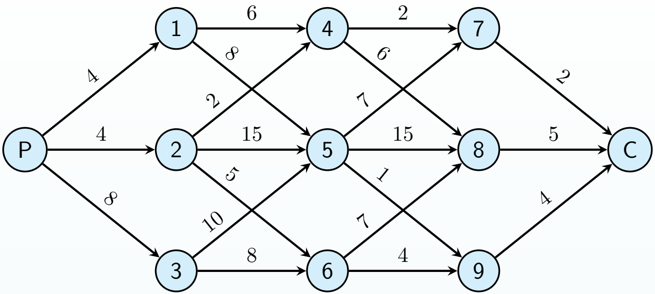


- Variation margin:
  $m(\mu^{\text{pru}}) = d_{\text{max}}(\mu^{\text{pru}}) - d_{\text{min}}(\mu^{\text{pru}}) = 47 - 30 = 17$

Stable path



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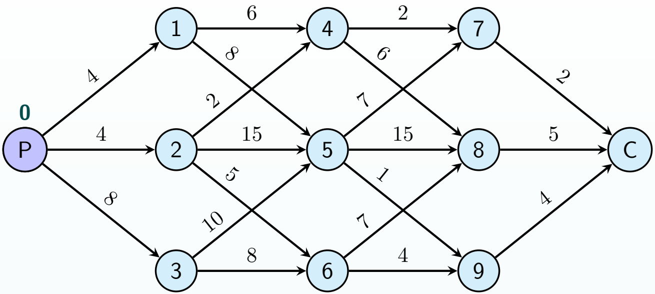
Stable path



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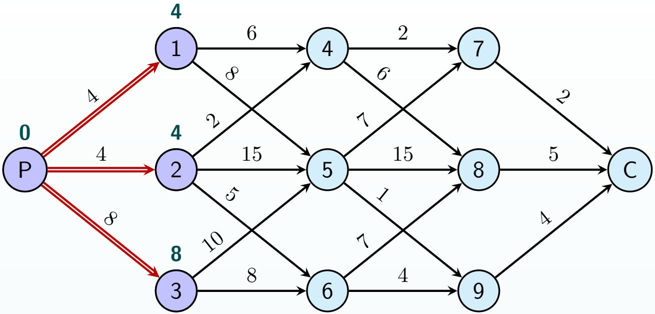
Stable path



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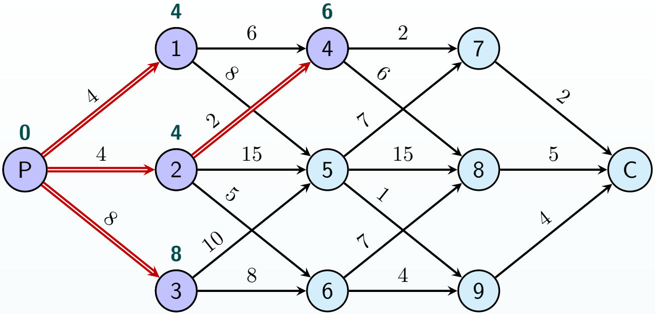
Stable path



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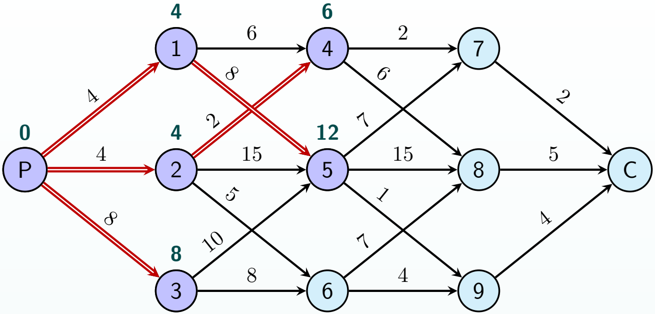
Stable path



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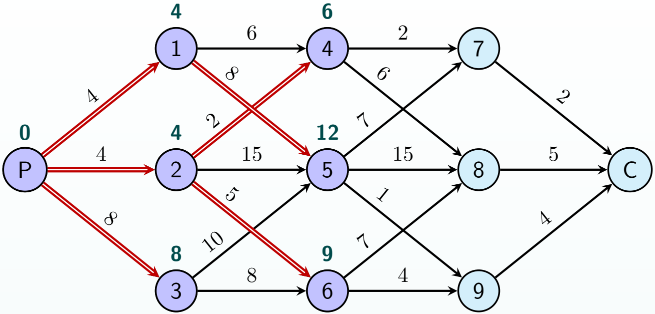
Stable path



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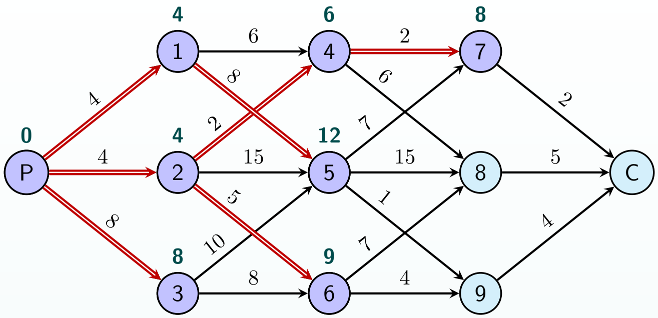
Stable path



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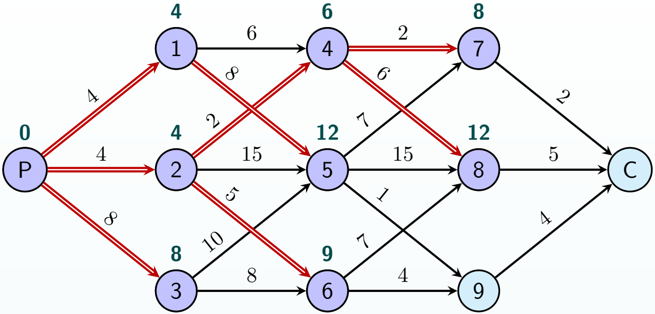
Stable path



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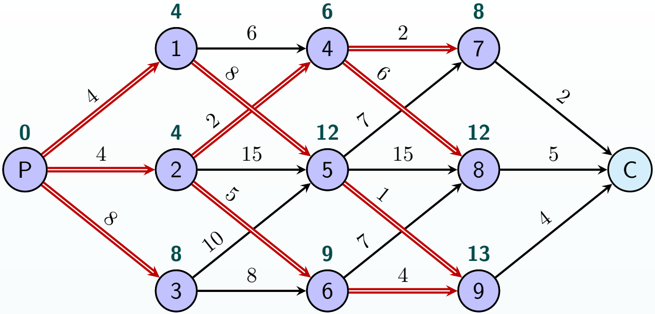
while $M \neq X$, do

select $x_j \in X \setminus M$ such as $P(x_j) \subset M$

$$\lambda_j \leftarrow \min_{i:x_i \in P_{x_j}} \{\lambda_i + v_{ij}\}$$

$$M \leftarrow M \cup \{x_j\}$$

Stable path



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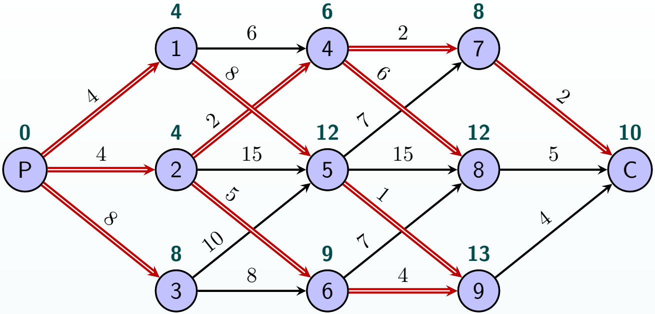
Stable path



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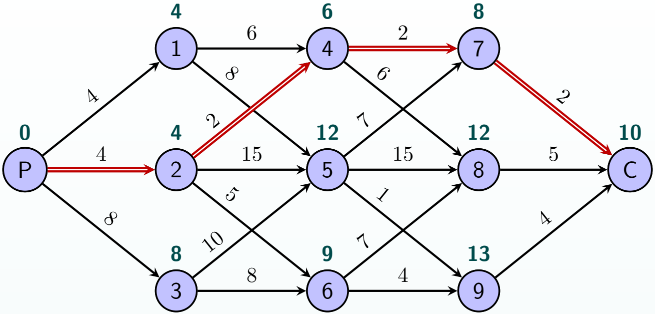
Stable path



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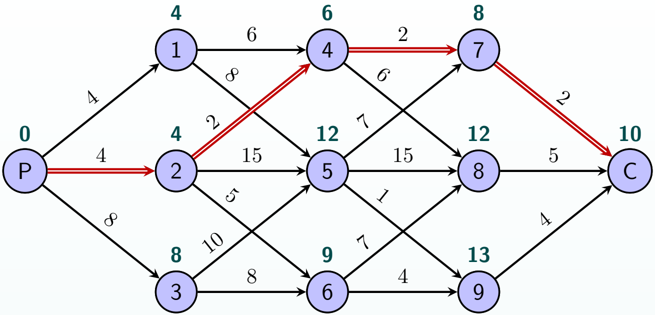
Stable path



- Variation margin:
  $m(\mu^{\mathsf{sta}}) = d_{\mathsf{max}}(\mu^{\mathsf{sta}}) - d_{\mathsf{min}}(\mu^{\mathsf{sta}}) = 54 - 44 = 10$

Conclusions

Path	**min**	**max**	μ
Optimistic	22	60	38
Careful	30	47	17
Stable	44	54	10

- Precautionary principle
- Assimilable to the complexity calculation

Graph Theory in OR – Application exercise 1: Crossing Paris
Damien Leprovost
March 6, 2015

permalink:
http://www.damien-leprovost.fr/enseignements/graphs.2015.ex1.pdf